Delay time measurements in a diffraction experiment: A case of optical tunneling

D. Mugnai,[1] A. Ranfagni,[1] and L. S. Schulman[1,2,3]
[1]Istituto di Ricerca sulle Onde Elettromagnetiche ''Nello Carrara''–CNR, Via Panciatichi 64, 50127 Firenze, Italy
[2]Physics Department, Clarkson University, Potsdam, New York 13699-5820
[3]Chemistry Department, Columbia University, New York, New York 10027
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Delay-time measurements in a diffraction experiment with microwaves have been performed both for the phase delay and for the group delay in the range of a few picoseconds to hundreds of picoseconds. The results obtained demonstrate that for evanescent modes below the cutoff frequency superluminal behavior was attained. [S1063-651X(97)01703-0]

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In an interesting paper published in 1949, Schaffner and Toraldo di Francia [1] reported experimental proof of the existence of evanescent (surface) waves generated by diffraction. Working at the scale of microwaves, they found that the experiment was easier and the results were more accurate, compared to those that had previously been performed in the optical range [2]. Microwaves of about 3 cm wavelength were used in connection with a grating made of metal strips. The period of the grating was chosen so that all the diffracted waves except the zero-order one were surface (evanescent) waves attenuated along the direction perpendicular to the grating. One of the first-order waves was transformed into an ordinary wave by refraction on a paraffin prism (see Fig. 1) and then revealed by means of a receiver. The measured power of the wave, as a function of the distance of the prism from the grating, gave the value of the attenuation constant of the surface wave as predicted by the theory.

We have performed a similar experiment with the aim of measuring the delay time for evanescent waves generated by diffraction. This represents a case of optical tunneling in which the duration of the process is still an open and debated question [3,4]. At the theoretical level there is complete acceptance of the idea that waves can never yield actual superluminal signals. However, increasingly imaginative experiments with evanescent waves, in tunneling processes [5–7] and in short-range propagation [8], have been probing this idea, for example, in forcing reexamination of just what constitutes a signal. As a result there has recently been extensive discussion of these issues [9–13] including the search for an appropriate interpretational model [14–16]. In this paper we report a particularly convenient way of generating evanescent waves—as indicated, it was in this way that exponential decay of the wave amplitude for evanescent waves was first demonstrated in a diffraction experiment.

The experimental setup is shown in Fig. 1 and includes, besides the grating and the prism, two horn antennas, one as a launcher and the other as a receiver. The latter is followed by a slotted waveguide where the signal picked up is combined with a reference signal derived from the generator. In this way we can make accurate phase measurements through the detection of the probe position corresponding to a minimum (which exactly indicates the opposition of phases of the two waves) of the amplitude of the resulting signal [17]. The results are shown in Fig. 2 where the probe position x is reported as a function of the distance d between the grating and the prism. The period of the grating is $a=3$ cm and the measurements were made at a frequency $\nu=9.24$ GHz, below the cutoff frequency $\nu_0=c/a=10$ GHz (c=vacuum light speed). The measurements were performed keeping all components at fixed positions; only the gap distance d was varied. (As a consequence the distance D between the grating and the launcher also changed, since $D_0 \equiv d+D=52$ cm was taken to be constant.) Of course we will also report on data (not shown in Fig. 2) at other frequencies.

Let us analyze the delay time. In the absence of grating, prism, etc., the total propagation time of the wave for the indicated distance would be D_0/c; thus, the time attributable to the traversal of the gap would be $d/c=D_0/c-D/c$. (For large D/λ, the velocity prior to reaching the grating is, to an excellent approximation, just c.) However, as the gap varies away from zero, the relative phase of the reference signal and the wave that does pass through the grating, etc., changes and the probe is moved until they again match. A probe displacement Δx means that the wave through the

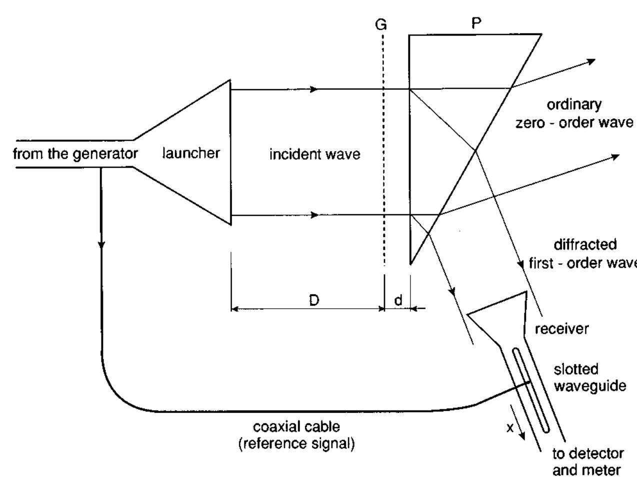

FIG. 1. The experimental setup consists of a grating G and a paraffin prism P separated by a gap whose width is d. Two horn antennas (the launcher and the receiver) allow accurate phase measurements by means of a slotted waveguide where the reference signal is combined with the signal picked up by the receiver.

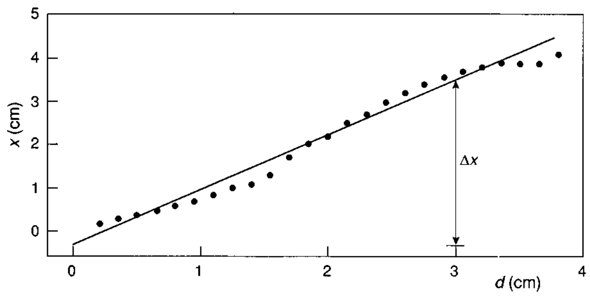

FIG. 2. Probe position x, relative to a constant phase value, as a function of gap width d for $\nu=9.24$ GHz. The probe position is approximately linear in the gap size and it is only the average slope of the curve that is ultimately used in Eq. (3). For this reason, our results are not sensitive to slight deviations from straight line behavior (the waviness in the curve) or to the zero position of the x variable. In practice, we used the variation Δx for a gap width $d=3$ cm.

prism has *gained* some time (with our sign convention). Therefore, using the phase velocity v_g within the slotted waveguide (where the probe is located), this yields for the time in the gap (as a function of frequency)

$$\tau_{\mathrm{ph}}(\nu)=\frac{d}{c}-\frac{\Delta x(\nu)}{v_g}, \tag{1}$$

where $v_g=c/\sqrt{1-(\lambda/2b)^2}$, λ is the free-space wavelength, and $b=22.86$ mm is the width of the waveguide. The quantity Δx is positive when the x position increases in distance away from the receiver horn (see Fig. 1).

By substituting v_g into Eq. (1), the phase delay can be rewritten as

$$\tau_{\mathrm{ph}}(\nu)=\frac{d}{c}\left[1-\frac{\Delta x(\nu)}{\Delta x'(\nu)}\right], \tag{2}$$

where, by denoting the wavelength in the waveguide, λ_g

$$\Delta x'(\lambda)=d\frac{\lambda_g}{\lambda}=\frac{d}{\sqrt{1-(\lambda/2b)^2}} \tag{3}$$

is the variation of the probe position in the waveguide corresponding to the variation d in free space. This means that if Δx turns out to be equal (or nearly equal) to $\Delta x'$, the phase delay in the gap is equal (or nearly equal) to zero. In the example of Fig. 2, referring to a frequency below the cutoff, the phase delay for a gap of 3 cm turns out to be $\sim$ 10 ps while the phase delay for the same distance in free space is 100 ps. This shows—as expected—that the phase delay of an evanescent wave is much shorter than the phase delay of a normal mode. The case of group delay is different. In free space this coincides with the phase delay, while in the gap—a dispersive medium—it is different. We will see in the following how the group delay can be deduced from measurements of the phase delay.

Before presenting our results, let us briefly consider what outcome is expected on the basis of an optical tunneling model. Although several models have been proposed, it is now widely accepted that the tunneling time is a complex quantity $\tau=\tau_\phi+i\tau_z$ with $\tau_\phi=(\partial\Delta\phi/\partial\omega)$ and $\tau_z=(\partial/\partial\omega)\ln T^{1/2}$, where $\Delta\phi$ is the phase variation across the barrier and T is the transmission coefficient [16]. Delay-time measurements are usually related to the real part, or phase time delay τ_ϕ, while the imaginary part can be deduced through attenuation measurements. The present experiments also support this point of view. The Büttiker model [18], which assumes the tunneling time to be $\sqrt{\tau_\phi^2+\tau_z^2}$, has not received experimental confirmation.

Adapting the quantum-mechanical approach to tunneling in a rectangular barrier [18] to the electromagnetic case [19,20], we can derive the phase variation $\Delta\phi$ across the gap region

$$\tan(\Delta\phi)=\frac{nk_1^2+k_2^2}{(1+n)k_1k_2}\tan(k_2d), \quad \nu\geqslant\nu_0, \tag{4}$$

where $k_1=2\pi\nu/c$ is the wave number before the grating, $k_2=(2\pi/c)\sqrt{\nu^2-\nu_0^2}$ the wave number associated with the first order diffracted waves in the gap, and $n=1.49$ is the refractive index of the paraffin where the wave number—beyond the gap—is $k_3=nk_1$.

Below the cutoff, for $\nu<\nu_0$, the diffracted waves become evanescent, attenuated along the x direction perpendicular to the grating as [1]

$$\exp\left(-k_1x\sqrt{\frac{\lambda^2}{a^2}-1}\right)=\exp\left(-\frac{2\pi x}{c}\sqrt{\nu_0^2-\nu^2}\right). \tag{5}$$

Under these conditions Eq. (4) becomes

$$\tan(\Delta\phi)=\frac{nk_1^2-\kappa_2^2}{(1+n)k_1\kappa_2}\tanh(\kappa_2d), \quad \nu<\nu_0, \tag{6}$$

where $\kappa_2=(2\pi/c)\sqrt{\nu_0^2-\nu^2}$ is the attenuation constant of the evanescent wave [Eq. (5)] in the gap [21]. This calculation ignores phase changes associated with diffraction and other semiclassical propagation phenomena as treated, for example, in [22]. We will include those phases below in analyzing the experimental data.

We are now in position to obtain the phase delay which is given by $\tau_{\mathrm{ph}}=\Delta\phi/\omega,\omega$ being $2\pi\nu$, and, consequently, we can derive the group delay [23,24]

$$\tau_{\mathrm{gr}}=\frac{d}{d\omega}\Delta\phi=\tau_{\mathrm{ph}}+\omega\frac{d}{d\omega}\tau_{\mathrm{ph}}. \tag{7}$$

The curves of τ_{ph} and τ_{gr} versus frequency are represented in Fig. 3 for a gap width of 3 cm. We note that below the cutoff frequency at 10 GHz not only is the phase delay significantly reduced with respect to the light-velocity limit ($d/c=100$ ps), but even the group delay is well below that limit. In other words, superluminal behavior is predicted for the ''signal'' velocity or, more precisely, for the *technical* signal velocity [11,13].

Phase measurements versus gap width d have been made at different frequencies. By fitting each data set by a straight line as in Fig. 2, we can determine, through Eq. (2), the phase delay for selected values of d. The results obtained for

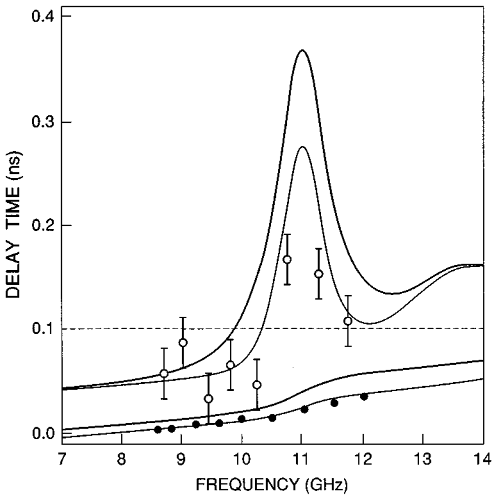

FIG. 3. Delay-time results and associated theoretical curves for a gap width of $d=3$ cm for which the corresponding time for light velocity propagation would be 0.1 ns (dashed line). Solid circles are the experimental phase-time delays obtained from phase measurements (Fig. 2). The two lower curves are the fitting of the experimental data (light line) and the phase-delay (heavy line) as predicted by the theoretical model $\tau_{\mathrm{ph}}=\Delta\phi/\omega$ where $\Delta\phi$ is given by Eqs. (4) and (6). Open circles with fiducial bars represent group-delay results derived from phase-delay data. The two upper curves are the group-delay model (heavy line) and the group delay deduced from the fitting curve below (light line).

$d=$ 3 cm are reported (full circles) in Fig. 3 as a function of the frequency. From these values we can deduce, according to Eq. (7) rewritten as

$$\tau_{\mathrm{gr}}=\tau_{\mathrm{ph}}+\nu\frac{\Delta\tau_{\mathrm{ph}}}{\Delta\nu}, \tag{8}$$

the group delay, represented by open circles with fiducial bars in the same Fig. 3. The fiducial bars have been estimated by the χ^2 criterion as $\sigma\approx(\Sigma_{i=1}^{N}\Delta_i^2/N)^{1/2}$ where Δ_i are the differences (residuals) between the theoretical and the N experimental values of τ_{ph} [25].

Since the calculation of the group delay involves the derivative of the measured quantities, it is reasonable to calculate first a smoothing of the data, and from that deduce the group delay. Ideally, if one had a theoretical description of the experiment which depended on one or two parameters, the measured data could be used to establish the values of those parameters. Then for the group velocity one would take the derivative of the phase velocity as given by a theoretical function dependent on the measured parameters (and error bars would reflect uncertainty in the values of the parameters). By contrast, a straightforward calculation of the derivative by taking differences of experimental values will exaggerate the normal variation of experimental output. Unfortunately, an exact theoretical calculation of the phase shift would be difficult for several reasons: (1) The prism is in the near field of the grating; (2) the wavelength is close to the critical value for the grating; (3) the slits themselves are of intermediate geometry, being neither infinite slits nor circles (each of which carries different phase factors [26]). We have therefore proceeded along both lines. We use a phenomenological model to motivate reasonable curve fitting and then use the fitted curve for calculation of the derivative. However, we also take the data and perform the most naive kind of derivative calculation. As will be seen below, the latter does not place τ_{gr} quite so deeply into the superluminal regime, although over a significant range even this leaves no doubt that the delays are less than would be obtained from velocity c.

The phenomenological model is based on the theoretical description of tunneling, Eq. (6) above. In addition to the phase change calculated there, there is a contribution due to the passage from forbidden to allowed propagation, Eq. (4). Such contributions have been calculated in a variety of situations (see [22]) although not for the intermediate-type geometry (nonrectangular, finite slits) of our experiment. We have therefore assumed that an additional phase change occurs, that it goes smoothly from zero to its full value (in practice we fit to a hyperbolic tangent), and we let the actual value of the phase shift be one of the parameters for the curve fitting.

The resulting data fit (with net phase change close to $2\pi/5$) is shown in the figure and it is the associated curve whose derivative is used in the calculation of group velocity. As indicated, the figure also displays (as circles) a calculation of the group delay that does not depend on any curve-fitting assumptions. For both the minimalist and the ''informed'' calculation and for frequencies below the cutoff, the group delay falls convincingly below that associated with the velocity of light. We have thus shown that for a range of accessible frequencies the group velocity is greater than c.

At this stage we prefer not to comment on the significance of this result for signals. We have begun to do experiments with this setup with pulses or other modulations of the carrier, rather than monochromatic microwaves. In this procedure one checks that the shape of a pulse is not substantially changed (otherwise ''group velocity'' would have little meaning). As reported briefly below, we have begun such measurements and they are so far consistent with our phase velocity measurements and the conclusions deduced from them. We feel, however, that at present, study of the phase delay is the most sensitive and reliable method.

When the delay measurements are made by using pulse modulation (such as a step function in which the transition has a duration of about 10 ns and the spectral width $\sim$100 MHz), the experimental set up of Fig. 1 turns out to be not very suitable when the measurements are made by varying d and D (the sum $d+D$ kept constant). Standing-wave effects, which give rise to the small undulation shown in Fig. 2, become amplified in the group-delay data (related to the derivative of the phase delay) versus the gap width so as to make more difficult the interpretation of the results. Presumably this could be overcome by keeping the distance D constant, but this would require a complete (nontrivial) modification of the experimental setup.

In addition, there is a modification in the results of delay measurements because of the so called ''speed-up effect'' [27,28]. This is due to the variation of the transmission coefficient in the frequency interval corresponding to the spectral width of the pulse (the barrier acts as a high-pass filter enhancing the transmission of the high-frequency components of the signal). This effect can be evaluated by noting

TABLE I. Attenuation constant κ_2 and semiclassical traversal time τ_s as deduced from amplitude measurements. The variation $\Delta\tau_\phi$ of the real delay time τ_ϕ with respect to the free-motion time L/c, as measured for $L=3$ cm by modulating at 10 MHz, is compared with the value deduced from phase measurements of the carrier (best-fit procedure). The reported error is consistent with the resolution of the lock-in amplifier.

ν (GHz)	κ_2 (cm^{-1})	τ_s (ps)	$\Delta\tau_\phi=\tau_\phi-L/c$ lock-in meas. (ps)	$\Delta\tau_\phi$ phase meas. (ps)
9.01	—	—	-97 ± 55	~-48
9.30	0.84	235	—	~-46
9.42	0.71	279	—	~-43
9.82	0.575	357	—	~-38
11.00	—	—	186 ± 55	~171

that the transmitted pulse turns out to be shifted towards the high-momentum values by an amount given by [28]

$$\frac{\Delta k}{k}=\frac{(\Delta s_k)^2}{2}\frac{c^2}{\omega}\frac{\partial}{\partial\omega}\ln T^{1/2}, \tag{9}$$

where Δs_k is the spectral width in momentum space. Since $(\partial/\partial\omega)\ln T^{1/2}$ is the imaginary component of the delay τ_z, and assuming $\Delta k/k\approx\Delta v_g/v_g=\Delta t/t, t$ being the duration of the complete travel in the experiment, we obtain an enhancement given by

$$\delta\tau=\left[\frac{t}{2}\left(\frac{\Delta\omega}{\omega}\right)^2\omega\right]\tau_z. \tag{10}$$

In our case we estimate the factor in parentheses to be on the order of 10^{-2} so that the measured delay should be shortened by about 1% of the imaginary component. This in turn can be assumed to be nearly coincident with the semiclassical time given by

$$\tau_s=\frac{2\pi\nu}{c^2}\frac{L}{\kappa_2}, \tag{11}$$

where the quantity κ_2 is obtained from amplitude measurements. Some results are reported in Table I together with the corresponding τ_s which are in rather good agreement with the value deduced from the theoretical value of $\kappa_2=(2\pi/c)\sqrt{\nu_0^2-\nu^2}$. Therefore, this interesting effect, even if appreciable, does not represent an important deviation from the expected results.

More reliable results, because of the smaller spectral extension with respect to the case of pulse modulation, have been obtained by measuring, using a lock-in amplification technique, the phase delay of a sinusoidal modulation that directly supplies the group delay or, more exactly, the variation $\Delta\tau_\phi$ with respect to the free-motion time L/c (see Table I). The modulation frequency ν_m was fixed at 10 MHz so that the spectral width of the signal is only 20 MHz. The sensitivity of this measure is not high since a delay of 100 ps corresponds to a dephasing of only 0.36°. Each result was obtained by a best fit of the data, relative to measurements of delay time versus the gap width d, and was affected by a large error as reported in Table I only for two frequency values because of the complexity of the procedure. Nevertheless, we can conclude that the results obtained in this way are in reasonable agreement with those derived from phase-delay measurements, deduced from Fig. 3 and reported in the last column of Table I. We emphasize that these data are preliminary and are not the principal results of the present paper. They are only included as a confirmation that the phase velocity results, and the conclusions we draw from them about group velocity, can be expected to be consistent.

Our purpose in this article has been to demonstrate a robust experimental setup in which group velocities greater than c occur. In particular the wave propagating through the grating gained on the order of 50 ps over a comparable light signal within 3 cm. This is a significant gain, one that is readily measurable. It is our expectation that this will allow exploration of the more far-reaching questions mentioned in the opening of this article, both by ourselves and others.

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